

# Allan Sandage and the cosmic expansion

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**Abstract** This is an account of Allan Sandage's work on (1) The character of the expansion field. For many years he has been the strongest defender of an expanding Universe. He later explained the CMB dipole by a local velocity of  $220 \pm 50 \text{ km s}^{-1}$  toward the Virgo cluster and by a bulk motion of the Local supercluster (extending out to  $\sim 3500 \text{ km s}^{-1}$ ) of  $450\text{--}500 \text{ km s}^{-1}$  toward an apex at  $l = 275$ ,  $b = 12$ . Allowing for these streaming velocities he found linear expansion to hold down to local scales ( $\sim 300 \text{ km s}^{-1}$ ). (2) The calibration of the Hubble constant. Probing different methods he finally adopted—from Cepheid-calibrated SNe Ia and from independent RR Lyr-calibrated TRGBs— $H_0 = 62.3 \pm 1.3 \pm 5.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

**Keywords** Cosmological parameter · Distance scale

## 1 Introduction

Edwin Hubble (1929) is generally credited for the discovery of the expansion of the Universe. But as so often in the case of fundamental discoveries, others had contributed. G. Lemaitre had published a value of the expansion rate (Hubble constant,  $H_0$ ) already in 1927, and H.P. Robertson once laconically told Sandage, Hubble found the expansion because I told him. In fact, Robertson had published his value of  $H_0$  already in 1928. Hubble's most astounding achievement is to have convinced the World of the expansion of the Universe with his brilliant monograph *The Realm*

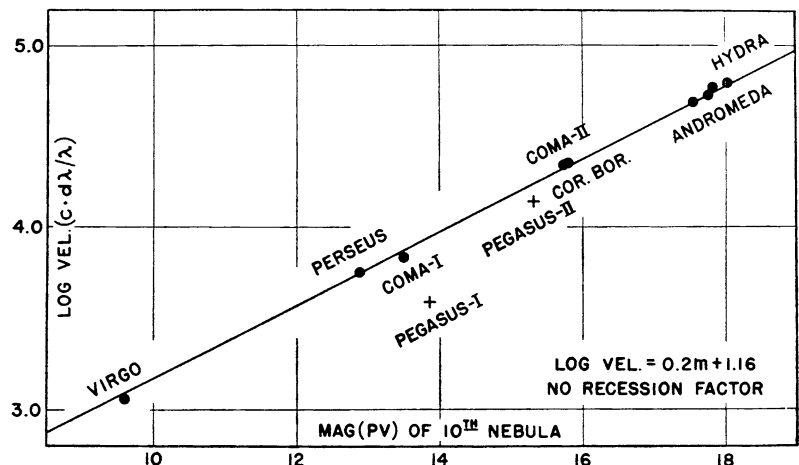


*of the Nebulae* (Hubble 1936a); he had by then much better cards than in 1929 because he had extended with the help of Milton Humason the log redshift-apparent magnitude diagram (Hubble diagram) to  $19,000 \text{ km s}^{-1}$  (Hubble and Humason 1934), but his value of  $H_0$  was still too high by a factor of roughly 8, and correspondingly his expansion age was impossibly short—a problem which he elegantly managed to bypass. Paradoxically Hubble began to question the reality of the expansion in the same year as his book appeared because he could not make sense of his galaxy counts. His doubts persisted until his death as evidenced in his Darwin Lecture—posthumously edited by Sandage—where Hubble (1953) showed a Hubble diagram including Humason's (Humason 1951) large-redshift clusters out to  $61,000 \text{ km s}^{-1}$  (Fig. 1) with the remark 'no recession factor (applied)', which means that he had corrected the galaxy magnitudes for a single factor of  $z$ , but not for the  $z^2$ -term required in any expanding model.

A definitive description of the expansion had to proceed along two lines. (1) The expansion field had to be mapped in different directions and out to truly cosmic distances—allowing for deceleration and/or acceleration—

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**Fig. 1** Hubbles last Hubble diagram with unrealistically small scatter and the remark no recession factor which documents his doubts about the expansion of the Universe

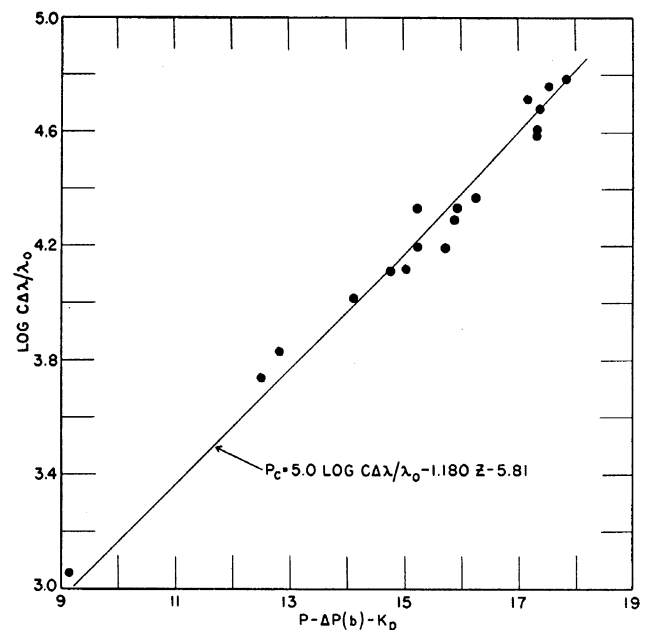


to test whether the expansion is linear, which means that it is observed as the same by any observer in the Universe.<sup>1</sup> (2) Only then it is meaningful to search for the cosmic value of  $H_0$ , which in turn would provide the first cosmological test, i.e. the expansion age of the Universe as compared with independent geological and astrophysical age determinations. Sandage has contributed to these two topics more than anybody else, although only about one fourth of his papers are devoted to them.

The remaining possibility that redshifts are *not* caused by the cosmic expansion has been disproved later by Sandage in a series of papers on the difficult Tolman test (Sandage 2010, and references therein) which requires that the surface brightness of a galaxy within a *metric* radius decreases with  $z^{-4}$ .

## 2 The character of the expansion field

The famous Humason et al. (1956) paper gave new support for an isotropic, expanding Universe. M. Humason and N. Mayall published in it the 630 galaxy redshifts of the combined Mount Wilson and Lick Observatory sample. The task of the theoretical analysis of the data fell upon Sandage. He homogenized the magnitudes, applied the first correct redshift-dependent K-corrections, and he showed Hubble diagrams for various subsamples. In particular he derived the Hubble diagram of 18 first-ranked cluster galaxies, where he applied corrections for luminosity evolution and the K-correction. From the upwards curvature of the Hubble line he concluded that the expansion is decelerated. In a subsequent paper Hoyle and Sandage defined the deceleration parameter  $q_0$  and derived a value of  $q_0 = 2.5 \pm 1$ , i.e. a decelerating Universe. It is interesting that this value flatly disagrees with  $q_0 = -1$ , the value required by Hoyle's Steady State



**Fig. 2** The Hubble diagram of first-ranked cluster galaxies from HMS. The curved Hubble line corresponds to a closed Universe with  $q_0 = 2.5$

model which he still maintained for a long time. The Humason et al. (1956) paper was the strongest support for an expanding Universe until 1962, when the Cosmic Microwave Background (CMB) was detected (Fig. 2).

In 1961 Sandage wrote a paper *The Ability of the 200-inch Telescope to Discriminate between Selected World Models*. It became the foundation of modern observational cosmology and made cosmology a quantitative science. He calculated the form of the Hubble diagram, the number of galaxies per apparent magnitude bin, and the diameter-redshift relation for a grid of different values of  $q_0$ , including  $q_0 = -1$ .

The large redshifts of Quasars were discovered in 1963. Sandage's rôle in the discovery is well described by Lynden-Bell and Schweizer (2012). Sandage also discovered the

<sup>1</sup> Note: linear expansion does not require a linear Hubble line.

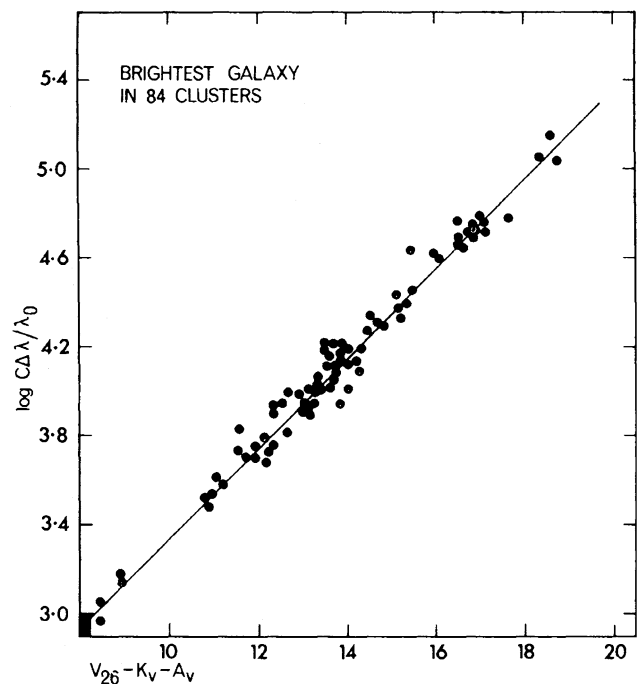


**Fig. 3** Allan Sandage in 1967. Under high pressure he had developed painful arthritis of his fingers, which is reflected in this picture

radio-quiet QSS (Sandage and Véron 1965; Sandage 1965). The discovery prompted widespread speculations about large non-cosmological redshifts. Sandage was appalled. At the IAU General Assembly in Prague in 1966 Sandage, together with Sir Martin Ryle, was the main speaker on the nature of large redshifts. He gave a flamboyant presentation, but some remained still unconvinced. Sandage felt an enormous pressure for the coming years, and he developed painful arthritis in his fingers, that later became dormant (Fig. 3).

### 2.1 The Hubble diagram of brightest cluster galaxies

During that time Sandage decided that the Hubble diagram of brightest cluster galaxies had to be carried to higher redshifts with the double purpose of determining  $q_0$  and to see how Quasars, radio galaxies, and N and Seyfert galaxies fitted into the picture. He single-handedly mounted a gigantic observing program for the identification, position, apparent magnitude and redshift of distant brightest cluster galaxies down to the limit of the 200-inch telescope. Precise positions were needed because the fainter objects could not be seen by eye, and the aperture photometry and photographic spectroscopy had to be done by blind offsets. He sometimes spent 14 hours without interruption in the narrow prime focus of the telescope, and he frequently changed, depending on the seeing conditions, the very heavy instruments during the night, which impaired his health. In total he invested more than 100 nights of the “Big Eye” on the program, that resulted in eight papers leaving no doubt that in order to explain the scatter in the Hubble diagram of various objects it was *not* necessary to invoke mysterious redshift, but that it was caused by the respective luminosity functions. By 1972 he had extended the Hubble diagram with a dispersion of  $\sim 0.3$  mag to  $z = 0.46$  as shown in Fig. 4. A formal solution for  $q_0$  gave  $q_0 \sim 1.0 \pm 0.5$ , yet excluding luminosity evolution. At the arrival of CCD detectors Westphal et al. extended the Hubble diagram to  $z = 0.75$ , but



**Fig. 4** Sandage's Hubble diagram of first-ranked cluster galaxies extending to  $z = 0.46$ . Lines for different values of  $q_0$  are shown. The box in the lower left marks the interval within which Hubble established the redshift-distance relation in 1929

without quoting a value of  $q_0$ , because in the mean time it had become clear that the light of E galaxies is dominated by red giants (Baldwin and Danziger 1973; Tinsley 1973; Tammann 1974) and that luminosity evolution has a decisive effect.

### 2.2 The Hubble diagram of Supernovae of type Ia (SNe Ia) and of clusters

Early Hubble diagrams of SNe I showed promise that they may become useful as standard candles, but the dispersion was still large (Kowal 1968; Tammann 1977, 1978, 1979; Branch and Bettis 1978). Nevertheless a scatter of less than 0.3 mag was suggested in a paper by Cadonau et al. (1985). The situation improved further with the spectroscopic separation of type Ia SNe from other subtypes (Branch 1986). This led to a luminosity dispersion of  $\sim 0.25$  mag (Tammann and Leibundgut 1990; Branch and Tammann 1992) which made SNe Ia strong competitors to brightest cluster galaxies as standard candles, in particular as they are presumably little affected by luminosity evolution. Their study was followed up by many authors, too numerous to be cited here, who increased the sample and improved the data (e.g. Hamuy et al. 1996). The suggestion of Phillips (1993) that the SNe Ia luminosity depended on the decline rate was initially questioned by Tammann and Sandage (1995), but later fully confirmed on the basis of more dis-

tant SNe Ia with reliable velocity distances in Parodi et al. (2000) and Reindl et al. (2005). The dispersion of the maximum magnitude was now reduced to 0.16 mag, or even less for the  $I$  magnitudes in dust-free E/S0 galaxies. (For the definition of the corrected maximum magnitudes see Sect. 3.3). The last paper in that series (Sandage et al. 2010, in the following SRT10) contains 246 SNe Ia with  $v_{\text{CMB}} < 30,000 \text{ km s}^{-1}$  (for the corrected velocities see Sect. 2.7). The sample is a compilation of  $\log v$  and  $m(\text{max})$  data from five large, overlapping sets of SNe Ia (cited in Sandage et al. 2010) which were homogenized by requiring that each set has to comply on average with the same, arbitrarily chosen value of  $H_0 = 60$ . The resulting Hubble diagram (Fig. 5 below) carries Sandage's expectation of a linearly expanding Universe down to scales of  $\sim 1200 \text{ km s}^{-1}$ . Others have carried the SNe Ia Hubble diagram to much higher redshifts and have thereby discovered dark energy (Riess et al. 1998; Perlmutter et al. 1999).

In addition to the SNe Ia good relative distances of 28 clusters with  $3000 < v < 10,000 \text{ km s}^{-1}$  have become available from the mean 21 cm-line width distances of about 25 individual cluster members per cluster (Masters 2008). Their Hubble line has a scatter of only 0.15 mag and shows no deviations from linear expansion; the line was fitted onto the line of SNe Ia by a shift in apparent modulus. They

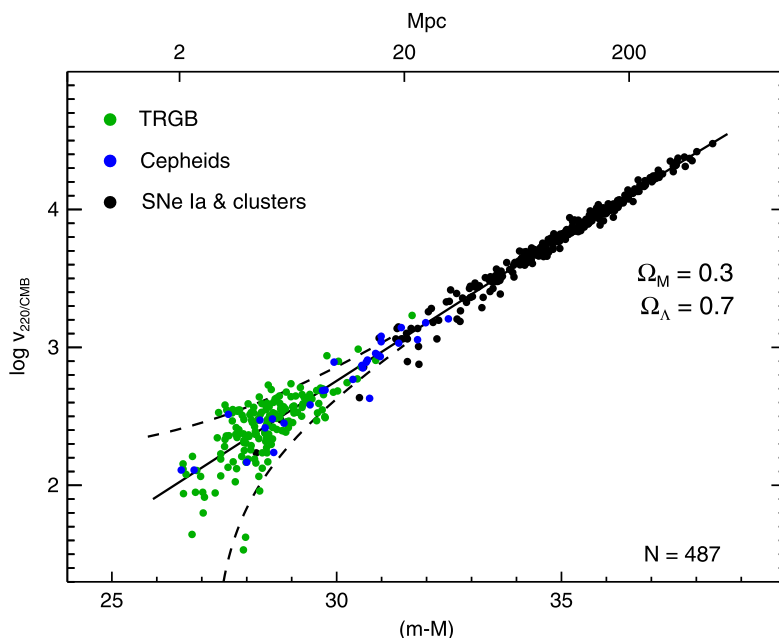
are included in Fig. 5. Also fitted onto the Hubble line are 11 clusters with good relative Fundamental-Plane (FP) distances from Jørgensen et al. (1996).

### 2.3 The Hubble diagram of Cepheids

Sandage (1986) traced the Hubble diagram also to lower velocities, using mainly Cepheid distances; the aim was to detect the perturbation of the Local Group on the local expansion field. Later, a sample of 29 Cepheids with a minimum distance of 4.4 Mpc was formed from the list of Saha et al. (2006) including a few additions. This sample, shown in Fig. 5 after normalization to the fiducial value of  $H_0 = 60$ , defines a Hubble diagram with a dispersion of 0.34 mag, much of which is caused by random velocities. An orthogonal fit to the data, assuming equal errors in magnitude and velocity, gives a slope of  $0.200 \pm 0.010$ , i.e. fully consistent with 0.2 for an isotropic Universe and  $z \ll 1$ .

### 2.4 The Hubble diagram of the tip of the red-giant branch (TRGB)

The absolute magnitude  $M^*$  of the TRGB has emerged as a powerful distance indicator, but, hardly reaching the Virgo cluster, its range is still limited—even more so than that of Cepheids. But locally the apparent TRGB magnitudes  $m^*$



**Fig. 5** The composite Hubble diagram of 176 galaxies with TRGB distances (green) and 30 galaxies with Cepheid distances (blue); the 246 SNe Ia and 35 clusters are shown in black. The distance moduli  $(m - M)$  are arbitrarily normalized to  $H_0 = 60$ . Galaxies with more than one distance determination are shown at the mean modulus. The velocities  $v_{220}$  are corrected for Virgocentric infall; if  $v_{220} >$

$3500 \text{ km s}^{-1}$  they are also corrected for the motion of the Local Supercluster toward the corrected CMB apex (see Sect. 2.7). The fitted, slightly curved Hubble line corresponds to a  $\Lambda$ CDM model with  $\Omega_M = 0.3$ ,  $\Omega_\Lambda = 0.7$ . The scatter is due to distance errors and to peculiar velocities. The effect of peculiar velocities of  $\pm 150 \text{ km s}^{-1}$  is shown by the two curved envelopes.

**Table 1** The final value of  $H_0$ 

Method	$v_{\text{med}}$	$N$	$H_0$	Ref.
TRGB	350	78	$62.9 \pm 1.6$	1
21cm line width	750	104	$59.0 \pm 1.9$	2
Cepheids	900	29	$63.4 \pm 1.8$	1
SNe Ia ( $v_{220} < 2000$ )	1350	20	$60.2 \pm 2.7$	2
SNe Ia ( $v_{\text{CMB}} > 3000$ )	7700	62	$62.3 \pm 1.3$	3
adopted			$62.3 \pm 1.3 (\pm 5.0)$	

References: (1) Tammann et al. (2008a); (2) Tammann et al. (2008b); (3) Sandage et al. (2006)

are ideal to trace the *mean* Hubble line because their large number compensates for the large scatter in  $\log v$  caused by the random velocities of nearby field galaxies.  $m^*$  magnitudes of 176 galaxies have been compiled (Tammann et al. 2008a, in the following TSR 08a) of which the nearer ones may be affected by the perturbation of the Local Group. The Hubble line with only the 78 more distant ones with  $m^* > 28.2$  has a slope of  $0.199 \pm 0.019$  in agreement with linear expansion. The sample of 176 TRGB is shown in Fig. 5, adjusted to the fiducial value of  $H_0 = 60$ .

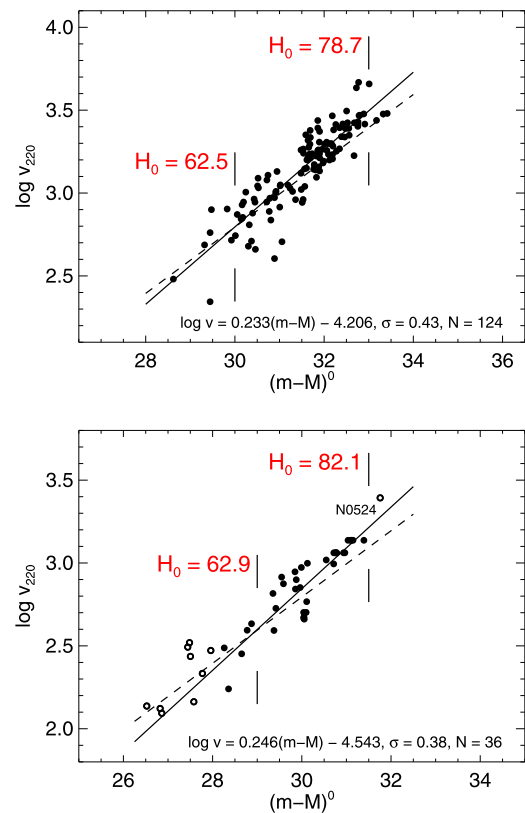
For the nearby Cepheid and TRGB distances it is important to note that all distances in Fig. 5 refer to the barycenter of the Local Group assumed to lie at two thirds of the way toward M 31 (Tammann et al. 2008a).

## 2.5 A composite Hubble diagram

The Hubble diagrams of SNe Ia (including 35 clusters), of the Cepheid distances, and of the TRGB magnitudes have been combined in a single diagram in Fig. 5 on the assumption that they comply to a common value of  $H_0$ . The question is to what extent the assumption is justified.

The intercept of the Hubble line of the SNe Ia has an error of  $\sigma(\log v) = 0.004$ . The corresponding error of the Cepheid Hubble line is 0.012. Hence the two partially overlapping Hubble lines can be connected within an error of  $\epsilon(\log v) = 0.013$  or  $\pm 0.07$  mag. The 78 TRGB galaxies outside 4.4 Mpc determine the intercept within  $\sigma(\log v) = 0.007$ ; merging them with the SNe Ia and Cepheid data causes hence an additional error of  $\epsilon(\log v) = 0.010$  or  $\pm 0.05$  mag. The combined fitting error between the nearest and the most distant objects is therefore  $\epsilon(\log v) = 0.016$  or 0.08 mag. This limits the variation of  $H_0$  with distance to about  $\pm 4\%$ . This value is now independent of any a priori assumption on  $H_0$ .

Additional evidence for the near constancy of  $H_0$  over the entire distance range comes from Table 1 below, where the value of  $H_0$  of the distant SNe Ia is given as well as the *independent* value of the nearby TRGB distances, including



**Fig. 6** (a) The Hubble diagram from SBF distances; data from Tonry et al. (2001). (b) The Hubble diagram from PNLf distances; data compiled from Ciardullo et al. (2002), Feldmeier et al. (2007), Herrmann et al. (2008). The dashed line stands for linear expansion

their statistical errors. From this follows a difference of  $H_0$  of only  $1 \pm 4\%$ .

The conclusion is that the cosmic value of  $H_0$  is the same as the mean local value at  $\sim 300 \text{ km s}^{-1}$  to within  $\lesssim 4\%$ .

## 2.6 Tests of various distance indicators against linear expansion

The linearity of the expansion allows to test the results of various distance indicators which—beyond  $300 \text{ km s}^{-1}$ —must yield mean values of  $H_0$  that are independent of distance. Examples are: the distances from surface brightness fluctuations (SBF) collected in Tonry et al. (2001) and the luminosity function of planetary nebulae (PNLF) (e.g. Ciardullo et al. 2002; Feldmeier et al. 2007; Herrmann et al. 2008). As seen in Fig. 6 they suggest that  $H_0$  increases beyond  $500 \text{ km s}^{-1}$  by more than 25% which is impossible in the light of Fig. 5. However, new work on the SBF method is promising; in any case the Fornax cluster modulus of  $31.54 \pm 0.02$  (Blakeslee 2012) is in good agreement with Sandage's value of  $31.62 \pm 0.10$  (Tammann et al. 2008a).

A large sample of relative  $D_n - \sigma$  distances of early-type galaxies out to  $10,000 \text{ km s}^{-1}$  has been published by Faber et al. (1989). The sample is not complete in any sense and



yields a Hubble diagram with a scatter of 0.7 mag. The corresponding incompleteness bias causes a seeming, but spurious increase of  $H_0$  with distance. The authors have therefore applied a bias correction which causes  $H_0$  to *decrease* by 10% out to the catalog limit, which suggests that the sample was somewhat overcorrected (Sandage et al. 2010, Fig. 3). A much smaller sample of related FP distances is apparently bias-free; it has been used in Sect. 2.2.

21 cm line width distances (Tully-Fisher relation) of inclined spiral galaxies have been determined by numerous authors. The crux of the method is its large intrinsic scatter of  $\sim 0.7$  mag, that is partially due to the difficult corrections for inclination and internal absorption. (The apparent scatter of *magnitude*-limited samples is of course smaller.) Distance determinations of field galaxies by some authors are therefore affected by incompleteness bias.

An attempt to correct 21 cm line distances for bias does not prove, but is consistent with linear expansion (Federspiel et al. 1994). A complete, distance-limited and therefore bias-free sample of 104 inclined field spirals can be defined out to only  $\sim 1000 \text{ km s}^{-1}$  (Tammann et al. 2008a); its large scatter does not allow to test for linearity. Useful, however, are the nearly complete spiral samples of the Virgo and UMa clusters (Tammann et al. 2008a).

The valuable cluster distances derived from many 21 cm line width data of a incomplete, but carefully bias-corrected sample of spiral members (Masters 2008) are mentioned already in Sect. 2.2.

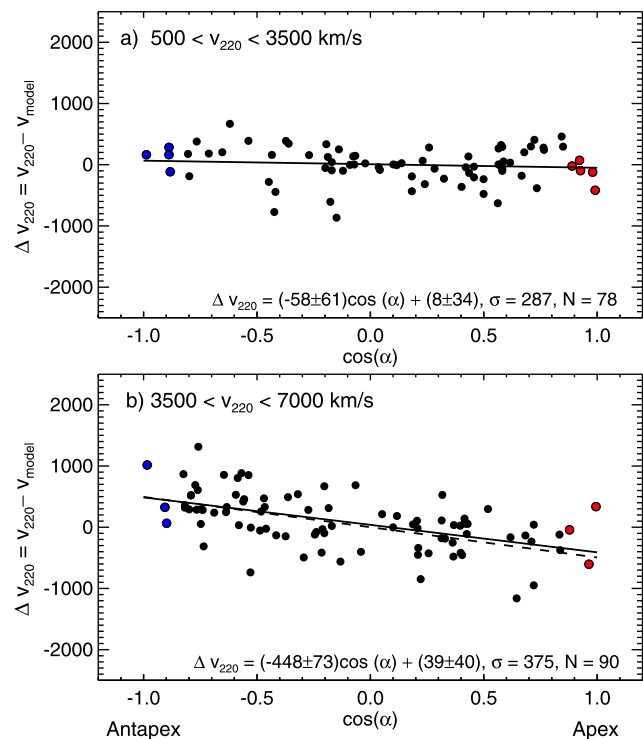
## 2.7 The local dipole velocity field

### 2.7.1 The Virgocentric infall vector of the Local Group

The first models of the velocity perturbations caused by the nearby Virgo cluster are due to Silk (1974) and Peebles (1976). Sandage and some of his collaborators authored several papers on the subject (e.g. Yahil et al. 1980; Sandage and Tammann 1982a, 1982b; Kraan-Korteweg 1985; Jerjen and Tammann 1993). Their value of the Virgocentric infall vector of the Local Group of  $220 \pm 50 \text{ km s}^{-1}$  (Tammann and Sandage 1985) encompasses most subsequent determinations. The value has been used to correct all velocities for a self-consistent Virgocentric infall model, which assumes a Virgo density profile of  $r^{-2}$  and, correspondingly, that the infall of individual galaxies scales with  $r^{-1}$ . An equation for the corrected velocities  $v_{220}$  is given in Sandage et al. (2006).

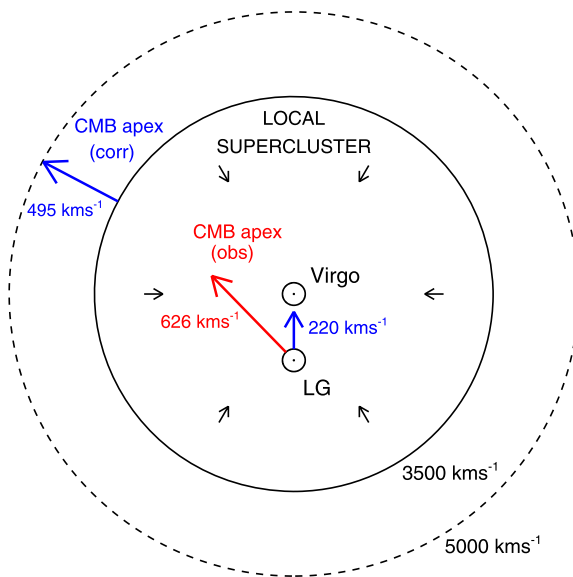
### 2.7.2 The motion relative to the Cosmic Microwave Background (CMB)

The observed velocity of the Local Group toward the CMB apex is the vector sum of the Virgocentric infall and of a still



**Fig. 7** (a) The velocity residuals  $\Delta v_{220}$  versus  $\cos \alpha$ , where  $\alpha$  is the angle between the object and the corrected CMB apex  $A_{\text{corr}}$ . The nearly horizontal line indicates that nearby objects with  $500 < v_{220} < 3500 \text{ km s}^{-1}$  have no significant systematic motion toward  $A_{\text{corr}}$ . (b) Same as (a), but for objects with  $3500 < v_{220} < 7000 \text{ km s}^{-1}$ . The slanted line indicates a bulk motion of the Local Supercluster of  $448 \pm 73 \text{ km s}^{-1}$  with respect to the Machian frame. Red (blue) points lie within  $30^\circ$  of the corrected apex (antapex)

larger velocity comprising a volume of unknown size. Taking the observed CMB velocity of  $626 \pm 30 \text{ km s}^{-1}$  (Hinshaw et al. 2007) toward an apex  $A_{\text{obs}}$  at  $l = 263.9$ ,  $b = 48.2$ , reducing it to the barycenter of the Local Group, and subtracting the Virgocentric infall one finds  $v_{\text{CMB}} = 495 \pm 25 \text{ km s}^{-1}$  towards the apex  $A_{\text{corr}}$  at  $l = 275 \pm 2$ ,  $b = 12 \pm 4$ . In order to determine the size of the co-moving volume a Hubble diagram was constructed as in Fig. 5, but now using the  $v_{220}$  velocities as ordinate. The residuals  $\Delta v_{220}$  from the resulting Hubble line were plotted versus  $\cos \alpha$ , where  $\alpha$  is the angle between the object and  $A_{\text{corr}}$ . After several trials the plot was divided into objects with  $v_{220} < 3500 \text{ km s}^{-1}$  (Fig. 7a) and into objects with  $3500 < v_{220} < 7000 \text{ km s}^{-1}$  (Fig. 7b). The residuals of the nearer objects show essentially no dependence on  $\cos \alpha$ . This means that they are at rest in first approximation relative to the Virgo cluster, once the Virgocentric velocities and the regular Hubble flow are subtracted. The inner volume emerges as the (truly contracting) Local Supercluster (Fig. 8). The objects with  $v_{220} > 3500 \text{ km s}^{-1}$  show a highly significant dependence on  $\cos \alpha$ . This reflects a bulk motion of the Local Supercluster of  $448 \pm 73 \text{ km s}^{-1}$  in good agreement with the expected



**Fig. 8** Schematic presentation of the local dipole velocity field

CMB value of  $495 \pm 25 \text{ km s}^{-1}$ . Most of the bulk motion must therefore be caused by the gravitational force, integrated over the whole sky, from the irregularly distributed masses between 3500 and  $<7000 \text{ km s}^{-1}$  (Sandage et al. 2010).

All velocities in this paper are corrected for Virgocentric infall and—in case of  $v_{220} > 3500 \text{ km s}^{-1}$ —for the adopted velocity of  $495 \text{ km s}^{-1}$  of the Local Supercluster toward the CMB apex  $A_{\text{corr}}$ .

### 3 The calibration of $H_0$

Hubble had based his galaxy distances on a few Cepheids, on brightest stars, and on the mean luminosity of galaxies. His result was  $H_0 = 525$ . Improvements of this value came slowly (for reviews see e.g. Sandage 1995, 1998, 1999; Tammann 2006). In 1948 Baade defined  $H_0$  as one of the prime targets for the new 200" telescope. But his seminal distinction between the young Population I and the old Population II (Baade 1952) was still based on observations with the 100" telescope. The new finding, that revealed the luminosity difference between RR Lyr stars and Cepheids, was confirmed by Sandage's thesis work (Sandage 1953) and reduced  $H_0$  by a factor of 2.

#### 3.1 Sandage's work on the calibration of $H_0$

In 1954 Sandage summarized the results from the first four years with the 200" telescope and concluded, mainly from a corrected magnitude scale, that  $125 < H_0 < 276 \text{ [km s}^{-1} \text{ Mpc}^{-1}]$ . He also found that some of Hubble's brightest stars are actually HII regions which are 2 magnitudes brighter; this and a new Cepheid distance of M 31 (Baade

and Swope 1954) led to  $H_0 = 180$  (Humason et al. 1956). In 1962 Sandage gave a review of  $H_0$  at the influential Santa Barbara Colloquium where he gave  $H_0 = 100$  as the mean of several authors, but his preferred value, considering also the size of HII regions, was  $H_0 = 75$ .

His well-known paper of Sandage (1970) *The search for two numbers ( $H_0$  and  $q_0$ )* started a new attack on  $H_0$ . It had begun already with the Cepheid distance of NGC 2403 (Tammann and Sandage 1968), the first galaxy outside the Local Group, and continued with a series of *Steps toward the Hubble constant* which used van den Bergh's (1960) luminosity classes of spirals in addition to the previous distance indicators. The result was  $H_0 = 57 \pm 3$  (Sandage and Tammann 1975 and references therein). This prompted a 10-year controversy with G. de Vaucouleurs (1977, and references therein) who had embraced a value of  $H_0 \sim 100$ . Subsequent papers of the series used also 21 cm line widths and the luminosity function of globular clusters giving, if anything, somewhat lower values (Sandage and Tammann 1995). Sandage (1988) derived from the old method of the luminosity classification of spirals a value of 42 which amused him because of the coincidence with "The Answer to the Ultimate Question" in Douglas Adams's fiction *The Hitchhiker's Guide to the Galaxy*.

After a pilot program to calibrate the luminosity of SNe Ia with brightest stars (Sandage and Tammann 1982b), Sandage formed a small team to observe with *HST* the Cepheids in galaxies with known SNe Ia. Previous attempts of a SNIa calibration depended mainly on an adopted Virgo cluster distance (e.g. Leibundgut and Tammann 1990), which itself is controversial. The program required—as described in the next three Sections—a re-evaluation of Cepheids as distance indicators, the luminosity calibration of SNe Ia, and the zero-point determination of the TRGB distances as an independent test.

#### 3.2 Cepheids

##### 3.2.1 The P-C and P-L relations of Cepheids

Sandage wrote about 50 papers on Cepheids. Already the first paper (Sandage 1958) brought a new physical understanding of the period-luminosity (P-L) relation of Cepheids which he derived from the theory of harmonic oscillations. He showed that the P-L relation must have intrinsic scatter, and that the relation is actually a period-luminosity-color relation.

A new P-L relation was constructed by superimposing the Cepheids of several external galaxies and by setting the zero point by means of up to 11 Cepheids that are members of Galactic clusters with known distances (Sandage and Tammann 1968, 1969, 1971).

A basic observational fact is that the colors of Cepheids depend on metallicity. This was first set out for the Galaxy

and SMC by Gascoigne and Kron (1965) and explained by Laney and Stobie (1986) not so much as a line blanketing effect, but as a temperature effect. The metallicity effect between Galactic, LMC, and SMC Cepheids becomes striking in their ( $B-V$ ) versus ( $V-I$ ) diagrams (Tammann et al. 2003, Fig. 7a&b, in the following TSR 03). A detailed analysis of model atmospheres reveals that the whole instability strip is shifted in the HR diagram by variations of the metal content (Sandage et al. 1999). If the ensuing period-color (P-C) relations are different then the pulsation equation requires that also the P-L relations must necessarily be metal-dependent (Sandage and Tammann 2008). Metal-specific P-C and P-L relations are therefore needed.

Only for three galaxies the necessary input data, i.e. intrinsic color and distance, are available: the Galaxy with  $[O/H]_{T_e} = 8.62$ , LMC with  $[O/H]_{T_e} = 8.36$ , and SMC with  $[O/H]_{T_e} = 7.98$ . The Galactic Cepheid colors are well determined (Fernie et al. 1995, Tammann et al. 2003); those in LMC and SMC have been derived in fields surrounding the Cepheids and independently of the Cepheids themselves (Udalski et al. 1999a, 1999b). The distances of LMC [ $(m - M) = 18.52$ ] and SMC [ $(m - M) = 18.93$ ] are known to better than  $\pm 0.10$  mag from a number of distance indicators that are independent of any assumption on the P-L relation of Cepheids (Tammann et al. 2008b, Table 6 & 7). The Galactic P-L relation relies on 33 Cepheids in Galactic clusters and associations and on 36 Cepheids with Baade-Becker-Wesselink distances; for the individual sources see Sandage et al. (2004). The two methods have been criticized by van Leeuwen et al. (2007), and the BBW method is blemished by the uncertain projection factor  $p$  (Nardetto 2012). Yet the steep slopes of the Galactic P-L relation from the independent cluster Cepheids and the BBW method (Fouqué et al. 2003) agree exceedingly well, and the steep slope is also observed in the metal-rich galaxies NGC 3351 and 4321 (Tammann et al. 2008b).

The finally adopted, only slightly revised P-C and P-L relations of the three calibrating galaxies are spelled out in Sandage's last paper (Tammann et al. 2011). The relations of LMC and SMC with their conspicuous breaks at  $\log P = 0.55$  and  $0.9$ , respectively, are compared here with the Galactic ones in Fig. 9.

Cepheids in five galaxies of very low metallicity like SMC, or even lower, yield particularly well to the application of the SMC P-C and P-L relations. The resulting distances agree with RR Lyr star and TRGB moduli to within  $\leq 0.05$  mag on average. This provides an interesting comparison of the independent distance scales of the young Population I and old Population II.

It has been proposed to use so-called Wesenheit pseudo-magnitudes  $\omega$  in order to deal with the problem of internal absorption. They are defined as  $\omega_V = m_V - R_V(B-V)$  or  $\omega_I = m_I - R_I(V-I)$ , where  $R_\lambda$  is the absorption-to-reddening ratio. *Intrinsic* color differences of Cepheids with

different metallicity are treated here—after multiplication with  $R_\lambda$ !—as an absorption effect. This leads of course to systematic distance errors.

### 3.2.2 Difficulties with Cepheids

The crux of Cepheid distances is that the internal absorption must be known which necessitates a priori assumptions about their (metal-dependent!) colors. Three problematic cases are mentioned in the following.

**M101** The 28 Cepheids in an outer metal-poor field of M101 (Kelson et al. 1996) give with the adopted P-C and P-L relations of LMC a small internal reddening of  $E(V-I) = 0.03$  and  $(m - M)^0 = 29.28 \pm 0.05$ . The 773 Cepheids (after exclusion of overtone pulsators) in two inner, metal-rich fields (Shappee and Stanek 2011) must be compared with the metal-rich P-C relation of the Galaxy resulting in excesses  $E(V-I)$  that increase with period. The absorption-corrected P-L relation, however, is significantly flatter than the Galactic P-L relation, but agrees well—in spite of higher metallicity—with the one of LMC. If the latter is adopted the modulus becomes  $29.14 \pm 0.01$ . Both of the two discrepant distance determinations are internally consistent inasmuch as either fulfills the important test that the *individual* Cepheid distances must not depend on the period. It seems to follow that the inner, metal-rich Cepheids are more luminous than assumed, or that the metal-poor, outer Cepheids are bluer and consequentially more absorbed than assumed.

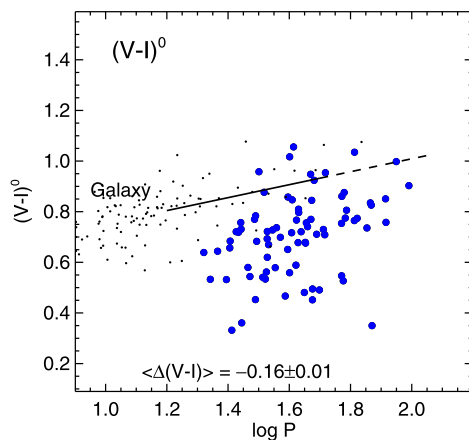
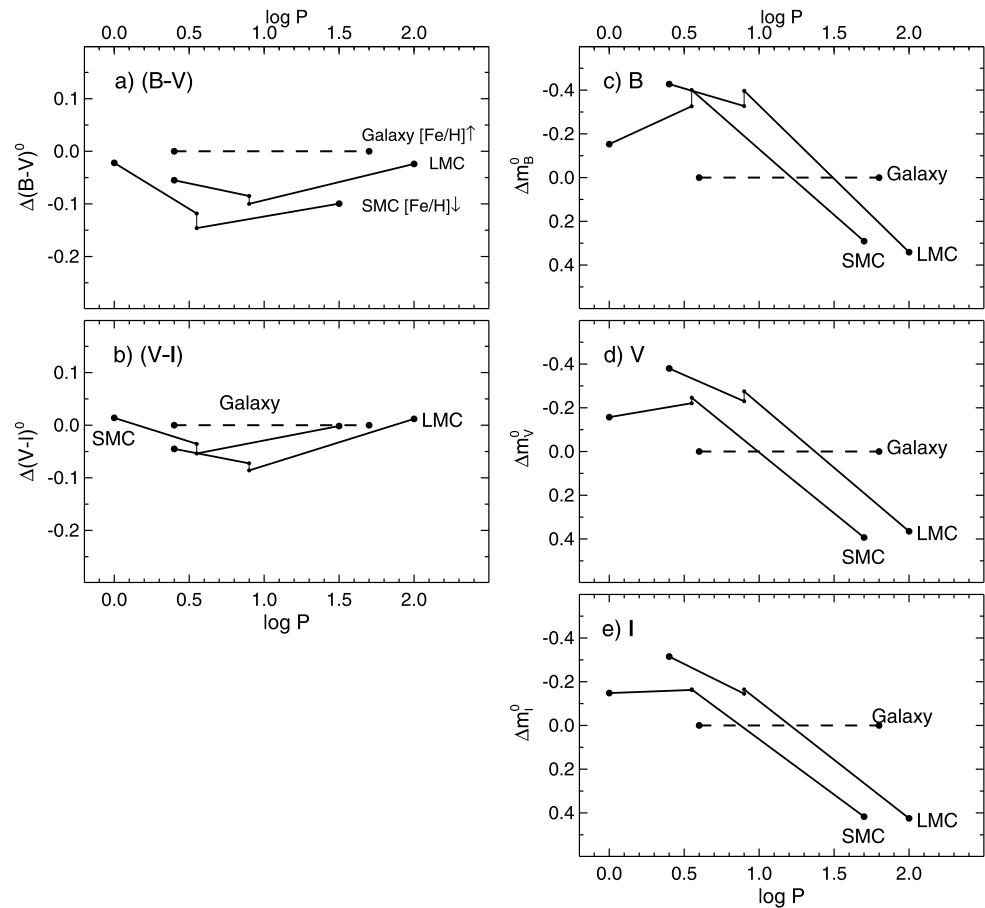
**NGC 4258** Macri et al. (2006) have provided 34 Cepheids in an outer, metal-poor field of NGC 4258 and 84 Cepheids in an inner, presumably metal-rich field. Díaz et al. (2000) and Kudritzky (2012), however, have shown that the inner field is almost as metal-poor as the outer field. The Cepheids in both fields should therefore be reduced with the P-C and P-L relations of LMC. One obtains then for the outer field  $E(V-I) = 0.03 \pm 0.03$  and  $(m - M)^0 = 29.47 \pm 0.02$  and for the inner field  $E(V-I) = 0.13 \pm 0.05$  and  $(m - M) = 29.18 \pm 0.02$ . The modulus discrepancy of  $\sim 0.3$  mag is worrisome. The Cepheids in the two fields, although of similar metallicity, do not seem to follow identical P-C and/or P-L relations.

It has been proposed to use NGC 4258 as a cornerstone for the distance scale because of its water maser distance of  $29.29 \pm 0.09$  (Herrnstein et al. 1999) and in spite of its remaining error. However, for other Cepheids, even if metal-poor, it is not clear whether they should be compared with the Cepheids in the outer or inner field.

**Blue Cepheids** The metal-rich Cepheids of NGC 1309 (Riess et al. 2009) have a P-C relation with unusually large



**Fig. 9** (a) and (b) The P-C relations in  $(B-V)$  and  $(V-I)$  of the Cepheids in the very metal-poor SMC and in the metal-poor LMC relative to the metal-rich Solar neighborhood. (c)–(e) The P-L relations in  $B$ ,  $V$ , and  $I$  of the Cepheids in SMC and LMC relative to the Solar neighborhood



**Fig. 10** The  $(V-I)$  colors of Cepheids in NGC 1309

scatter and are in  $(V-I)$ , even without a reddening correction, 0.16 mag bluer on average than the presumably equally metal-rich Galactic Cepheids (Fig. 10). In fact they are by far the bluest long-period Cepheids known. The effect went unnoticed because of the use of the Wesenheit pseudomagnitudes. The Cepheids constitute a new class. Without knowledge of their true P-C and P-L relations it is of course not possible to determine their distances. The case is alarming

because also the Cepheids of NGC 3021 (Riess et al. 2009) appear to be too blue by  $0.07 \pm 0.03$ , and additional intrinsically blue Cepheids may appear red because of reddening in their parent galaxies.

These examples and particularly the ultra-blue Cepheids in NGC 1309 suggest that an additional, hidden parameter influences the properties of Cepheids. It has been discussed in the literature whether the Helium content could be the culprit (e.g. Marconi et al. 2005; Bono et al. 2008).

More recently infrared  $H$ -magnitudes of Cepheids in a few galaxies have become available. They are less sensitive to absorption and metal lines, but this does not prove them to be free of other effects. Additional data are needed for an independent test.

### 3.3 The luminosity calibration of SNe Ia

Different authors have homogenized SNIa data in different ways. The particulars of the method of Sandage's team are laid out in Reindl et al. (2005). In short, their sample excludes known spectroscopically peculiar SNe Ia. The SN colors  $(B-V)$  and  $(V-I)$ , corrected for Galactic reddening, are defined as the difference of the  $K$ -corrected magnitudes  $m_B^{\max}$ ,  $m_V^{\max}$ , and  $m_I^{\max}$ . The intrinsic colors  $(B-V)$

and  $(V-I)$  as well as the color  $(B-V)^{35}$ , 35 days after  $B$  maximum, are determined from (dust-free) SNe Ia in E, S0 galaxies and from outlying SNe Ia in spirals with a slight dependence on  $\Delta m_{15}$ . Corresponding corrections for internal absorption are applied throughout adopting a reddening-to-absorption ratio of  $R_B = 3.65$  as required by the data (instead of the canonical value of 4.1). The decline rate  $\Delta m_{15}$  is defined as usual as the brightness decline in magnitudes over the first 15 days after  $B$  maximum. The corrected colors, normalized to  $\Delta m_{15} = 1.1$  become  $(B-V) = -0.02$ ,  $(V-I) = -0.27$ , and  $(B-V)^{35} = 1.11$ .

Also the absolute magnitudes based on velocity distances show a pronounced dependence on  $\Delta m_{15}$ . The additional dependence on galaxian type disappears when the magnitudes are normalized to  $\Delta m_{15} = 1.1$ . The 62 SNe Ia, corrected for Galactic and internal absorption and normalized to  $\Delta m_{15} = 1.1$ , in the well populated range of the Hubble diagram between 3000 and 20,000 km s<sup>-1</sup> have mean absolute magnitudes of  $M_B = -19.57$ ,  $M_V = -19.55$ , and  $M_I = -19.28$  as judged from their velocity distances assuming  $H_0 = 60$ . The statistical error of the mean absolute magnitudes is only 0.02 mag.

The *HST* Supernova Project (Sandage et al. 2006) gives for ten SNe Ia with metallicity-corrected Cepheid distances weighted luminosities of  $M_B = -19.49 \pm 0.07$ ,  $M_V = -19.46 \pm 0.07$ , and  $M_I = -19.22 \pm 0.06$  in the system of Reindl et al. (2005). These values compared with those in the previous paragraph yield a mean value of  $H_0 = 62.3 \pm 1.3$ . The statistical error depends almost entirely on the calibration and not on the definition of the Hubble line. Correspondingly the systematic error of  $\pm 5$  (estimated) is dominated by errors of the Cepheid distances.

To emphasize the difference between the SN magnitudes as defined here and those used by other authors it is noted that, for instance, the apparent SN magnitudes as reduced by Jha et al. (2007) are fainter by  $\Delta m_V = 0.13$  mag on average than here. This is purely the result of the definition of the corrected value of  $m_{\max}$ .

### 3.4 The calibration of the tip of the red-giant branch

The fascinating property of the TRGB is that its calibration is straightforward and that the maximum brightness of red giants is limited by basic physics. Particularly stable is the near-infrared maximum magnitude  $I^*$  of red giants in old, metal-poor halo populations (Da Costa and Armandroff 1990), where also internal absorption poses a minimum problem. The practical difficulty is the observational determination of the upper limit  $I^*$ , which requires a sufficiently large sample and the separation of AGB stars. For the history and model calculations of the TRGB see Salaris (2012).

The obvious way to calibrate the TRGB is by RR Lyr stars. Sandage has devoted 50 papers to these stars, exploring their classification, metal content, evolution, age etc. His last metal-dependent, non-linear luminosity calibration is  $M_V(\text{RR}) = 1.109 + 0.600[\text{Fe}/\text{H}] + 0.140[\text{Fe}/\text{H}]^2$ , i.e.  $M_V(\text{RR}) = 0.52$  mag at  $[\text{Fe}/\text{H}] = -1.5$  (Sandage and Tammann 2006). This calibration has been applied to 24 galaxies for which RR Lyr magnitudes are available in the literature as well as TRGB magnitudes  $I^*$  (for the many original sources see Tammann et al. 2008b). The combination of the RR Lyr moduli with the corresponding apparent  $I^*$  magnitudes yields the absolute magnitudes  $M_I^*$ . The mean magnitude of the sample—with a mean color of  $(V-I)^* = 1.6$  or  $[\text{Fe}/\text{H}] = -1.5$  and omitting two deviating cases—is  $M_I^* = -4.05 \pm 0.02$ , where the dispersion is 0.08 mag (Tammann et al. 2008b). Exactly the same value has been found by Sakai et al. (2004) from globular cluster distances, and by Rizzi et al. (2007) from fitting the Horizontal Branch (HB) of five galaxies to a metal-corrected HB with a known trigonometric parallax. Also the model luminosities of Bergbusch and Vandenberg (2001) and Salaris (2012) are close to the empirical calibration.

The question to what extent  $M_I^*$  depends on the metallicity has repeatedly been discussed in the literature. Most authors agree that the luminosity does not change by more than  $\pm 0.05$  mag over the relevant range of  $1.4 < (V-I)^* < 1.8$  or  $-2.0 < [\text{Fe}/\text{H}] < -1.2$  (see Fig. 1 in Tammann et al. 2008b).

The adopted TRGB moduli of 17 galaxies, for which also Cepheid moduli are available (listed in Tammann et al. 2008a), reveal that they are larger by a marginal amount of  $0.05 \pm 0.03$ , than the Cepheid moduli. This shows that Sandage's TRGB and Cepheid distances, although fully independent, are in satisfactory agreement. The dispersion of the differences of  $\sigma = 0.13$  mag suggests that the individual TRGB and Cepheid distances carry random errors of less than  $\sim 0.1$  mag.

The mean  $M_I^*$  magnitudes of 240 galaxies of the many values in the literature have been averaged and normalized to the above calibration. The resulting distances are listed in Tammann et al. (2008a). The subsample of 78 galaxies more distant than 4.5 Mpc gives  $H_0 = 62.9 \pm 1.6$ .

In the future it will be important to extend the range of TRGB distances beyond 1000 km s<sup>-1</sup> in order to tie them even tighter to the cosmic expansion field and/or to provide an independent luminosity calibration of SNe Ia. First attempts have been made (Tammann et al. 2008b; Mould and Sakai 2009).

### 3.5 Sandage's last value of the Hubble constant

Sandage has persued the calibration of  $H_0$  for almost 60 years. It was his aim from the beginning to base his distance



scale on two independent pillars, i.e. on Population I and Population II objects, and he spent about equal efforts on either route. The distance scale of the former depends heavily on Cepheids, whereas that of the Population II relies mainly on RR Lyr stars. The determination of Cepheid distances has become more involved because of the metal dependence of the P-C and P-L relations, accentuated by the corresponding problems of internal absorption and other unexplained effects—in particular of the more metal-rich Cepheids (see Sect. 3.2.2). Hence the need for a second pillar has become even more urgent. The direct comparison of Cepheids and RR Lyr stars is unprofitable because of the paucity of galaxies with reliable data on both distance indicators. But here the RR Lyr-calibrated TRGB distances jump in, which offer ample comparison with Cepheid distances (Tammann et al. 2008a, Table 9). More important yet was for him that the—admittedly still local—value of  $H_0$  from the TRGB is the same within the statistical errors as that from Cepheids and Cepheid-calibrated 21 cm distances and SNe Ia as summarized in Table 1.

Some months before Sandage's death Reid et al. (2010) published a paper combining the catalog of luminous red galaxies (LRG) from the Sloan Digital Sky Survey DR7 with the 5-year WMAP data and the Hubble diagram of the SNe Ia Union Sample to find a value of  $H_0 = 65.6 \pm 2.5$  on the assumption of a  $\Lambda$ CDM model.

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